hep-th/9412029
IPM-94-071
IASBS 94-6
TUDP 94-2
SUTDP 94-73-11

A Triangular Deformation of the two Dimensional Poincaré Algebra

M. Khorrami [1,2,3,*], **A. Shariati** [1,4], **M. Abolhasani** [1,4], **A. Aghamohammadi** [1,5]

[1] *Institute for Studies in Theoretical Physics and Mathematics, P.O.Box 5746, Tehran 19395, Iran.*

[2] *Department of Physics, Tehran University, North-Kargar Ave. Tehran, Iran.*

[3] *Institute for Advanced Studies in Basic Sciences, P.O.Box 159, Gava Zang, Zanjan 45195, Iran.*

[4] *Department of Physics, Sharif University of Technology, P.O.Box 9161, Tehran 11365, Iran.*

[5] *Department of Physics, Alzahra University, Tehran 19834, Iran*

[*] *E-Mail: mamwad@irearn.bitnet*

Abstract

Contracting the h-deformation of $\mathrm{SL}(2,\mathsf{R})$, we construct a new deformation of two dimensional Poincaré algebra, the algebra of functions on its group and its differential structure. It is also shown that the Hopf algebra is triangular, and its universal R matrix is also constructed explicitly. Then, we find a deformation map for the universal enveloping algebra, and at the end, give the deformed mass shells and Lorentz transformation.

1 Introduction

Deformations of the Poincaré group have recieved considerable interest in recent years [1-3]. These deformations may be considered as the authomorphisms of the quantum deformed space-time. Although a general theory of quantum deformations of inhomogenous quantum groups is lacking, there are attempts for constructing these non-semisimple groups [1-5]. As we know, there are at least three distinct deformations of the Poincaré group in four dimensions. The first is q-$\mathsf{P}(4)$, introduced by Ogievetsky et al. [1]. The second one is κ-$\mathsf{P}(4)$, introduced by Lukierski et al. [2, 3]. This κ-$\mathsf{P}(4)$ is obtained by a contraction

method. There exist another deformation which is constructed in [6]. A natural question to ask, is the number of distinct deformations of $\mathsf{P}(4)$.

It seems easier to consider the two dimensional case. In fact, the first step for constructing inhomogeneous quantum groups is related to the q-analog of two dimensional Euclidean group [7-9]. There exist two distinct deformations of E(2), both of which can be obtained from $\mathrm{SU}_q(2)$ by a contraction procedure [7, 9, 10]; neither of them has a universal R mtrix. In two dimensions, however, there exist two distinct deformations of SL(2), the well-known q-deformation, and the h-deformation [11-19]. The h-deformation itself can be obtained from the q-deformation by a contraction procedure [19].

In this article we obtain a new deformation of $\mathsf{P}(2)$ from the contraction of the h-deformation of $\mathrm{SL}(2,\mathsf{R})$. First, we construct the deformed universal enveloping algebra $\mathrm{U}_\mu\left(\mathsf{p}(2)\right)$, the deformed algebra of functions on the $\mathsf{P}(2)$, and its differential calculus. Then, we show that $\mathrm{U}_\mu\left(\mathsf{p}(2)\right)$ has a universal R matrix, and explicitly construct the universal R matrix. It is seen that this Hopf algebra is triangular. This is the distinguishing feature of this deformation of $\mathsf{P}(2)$, note that the other deformations of E(2) don't have universal R matrices. We also give a deformation map which relates the commutation relations of $\mathrm{U}_\mu\left(\mathsf{p}(2)\right)$ to their classical counterparts. This can be used to study the representation theory of $\mathrm{U}_\mu\left(\mathsf{p}(2)\right)$. Finally, we give the deformed mass shells and finte Lorentz transformation for energy and momentum in light cone coordinates.

2 μ-Deformation of the two dimensional Poincaré group

The h-deformation of $\mathrm{SL}(2,\mathsf{R})$ has the following structure. The deformed algebra of functions, $\mathrm{SL}_h(2)$ is generated by the enteries of the matrix

$$\mathrm{M} := \begin{pmatrix} a & b \\ c & d \end{pmatrix}. \tag{1}$$

and has the following Hopf algebra structure.

$$\begin{array}{lll} [a,c]=hc^2 & [d,b]=h(D-d^2) & [a,d]=h(dc-ac) \\ [d,c]=hc^2 & [b,c]=h(ac+cd) & [b,a]=h(a^2-D) \end{array} \tag{2}$$

$$\begin{array}{ll} \Delta a = a\otimes a + b\otimes c & \Delta b = a\otimes b + b\otimes d \\ \Delta c = c\otimes a + d\otimes c & \Delta d = c\otimes b + d\otimes d \end{array} \tag{3}$$

$$\begin{array}{ll} S(a)=d+hc & S(b)=-b-h(a-d)+h^2c \\ S(c)=-c & S(d)=a-hc \end{array} \tag{4}$$

$$\begin{array}{ll} \epsilon(a)=1 & \epsilon(b)=0 \\ \epsilon(c)=0 & \epsilon(d)=1 \end{array} \tag{5}$$

$$D = \det{}_h\mathrm{M} := a\,d - c\,b - h\,c\,d = 1 \tag{6}$$

The deformed universal enveloping algebra is generated by three generators X, Y, and H. The Hopf algebra structure, as first introduced by Ohn [14], is the following.

$$
\begin{aligned}
&[H,X] = \frac{2}{h}\sinh hX \\
&[H,Y] = -\{Y, \cosh hX\} \\
&[X,Y] = H
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
&\Delta X = X\otimes 1 + 1\otimes X \\
&\Delta Y = Y\otimes \exp(hX) + \exp(-hX)\otimes Y \\
&\Delta H = H\otimes \exp(hX) + \exp(-hX)\otimes H
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
&S(X) = -X \\
&S(Y) = -\exp(hX)\, Y \exp(-hX) \\
&S(H) = -\exp(hX)\, H \exp(-hX)
\end{aligned}
\tag{9}
$$

$$
\epsilon(X) = 0 \quad \epsilon(Y) = 0 \quad \epsilon(H) = 0 \tag{10}
$$

These two Hopf algebras are dual. The duality is

$$
\begin{aligned}
&\left\langle H, \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\
&\left\langle Y, \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right\rangle = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \\
&\left\langle X, \begin{pmatrix} a & b \\ c & d \end{pmatrix} \right\rangle = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}
\end{aligned}
\tag{11}
$$

Now we use the following contraction to obtain a new deformation of $\mathsf{P}(2)$ and $\mathrm{U}\left(\mathsf{p}(2)\right)$. For $\mathrm{U}_h\left(\mathsf{sl}(2,\mathsf{R})\right)$ we define

$$
P^+ := \epsilon X, \qquad P^- := \epsilon Y \qquad K := \frac{1}{2}H \qquad \mu := \epsilon h^{-1} \tag{12}
$$

As we shall see, $P^{\pm}$, are the deformed momenta in the light cone coordinates, and K is the deformed Lorentz boost. In the $\epsilon \to 0$ limit we get the following Hopf algebra which is a deformation of $U(\mathsf{P}(2))$.

$$
\begin{aligned}
&[K,P^+] = \mu \sinh(\tfrac{P^+}{\mu}) \\
&[K,P^-] = -P^- \cosh(\tfrac{P^+}{\mu}) \\
&[P^+,P^-] = 0
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
&\Delta P^+ = P^+\otimes 1 + 1\otimes P^+ \\
&\Delta P^- = P^-\otimes \exp(\tfrac{P^+}{\mu}) + \exp(-\tfrac{P^+}{\mu})\otimes P^- \\
&\Delta K = K\otimes \exp(\tfrac{P^+}{\mu}) + \exp(\tfrac{P^+}{\mu})\otimes K
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
&S(P^+) = -P^+ \quad S(P^-) = -P^- \quad S(K) = -K \\
&\epsilon(P^+) = 0 \qquad\quad\;\; \epsilon(P^-) = 0 \qquad\quad\; \epsilon(K) = 0
\end{aligned} \tag{15}
$$

To obtain the deformed algebra of functions, we use the known duality (11), from which we conclude that the corresponding contraction in the algebra of functions is

$$
\alpha := a \qquad \beta := \epsilon^{-1} b, \qquad \gamma := \epsilon^{-1} c \qquad \delta := d \qquad \mu := \epsilon h^{-1}. \tag{16}
$$

Expressing the commutation relations of $\mathrm{SL}_h(2, \mathsf{R})$ in terms of these new generators, and going to the $\epsilon \to 0$ limit, we obtain

$$
\begin{aligned}
&[\alpha, \beta] = \mu^{-1}(1 - \alpha^2) \quad [\alpha, \delta] = 0 \\
&[\delta, \beta] = \mu^{-1}(1 - \delta^2) \quad\; [\gamma, \cdots] = 0
\end{aligned} \tag{17}
$$

The four generators of $\mathrm{SL}_h(2, \mathsf{R})$ are not independent since the determinent of M must be equal to one. This leads to

$$
\det{}_\mu \mathrm{M} = \alpha\delta = 1 \qquad \Rightarrow \qquad \delta = \alpha^{-1} \tag{18}
$$

The calculations for the co-product, co-unity, and antipode is straightforward. For completeness, we write them here.

$$
\begin{aligned}
&\Delta\alpha = \alpha \otimes \alpha \qquad\qquad\quad\;\; \Delta\beta = \alpha \otimes \beta + \beta \otimes \delta \\
&\Delta\gamma = \gamma \otimes \alpha + \delta \otimes \gamma \quad \Delta\delta = \delta \otimes \delta
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
&S(\alpha) = \delta \qquad S(\beta) = -\beta - \mu^{-1}(\alpha - \delta) \\
&S(\gamma) = -\gamma \quad S(\delta) = \alpha
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
&\epsilon(\alpha) = 1 \quad \epsilon(\beta) = 0 \\
&\epsilon(\gamma) = 0 \quad \epsilon(\delta) = 1
\end{aligned} \tag{21}
$$

The differential structure of $\mathsf{P}_\mu(2)$ can be obtained in the similar way. One can use differential structure of $\mathrm{SL}_h(2)$, applying the same contraction proccess in the level of differentials too. To construct differential structure one should use the following relations [20, 21, 16]

$$
\mathrm{R}\mathrm{d}\mathrm{M}_1\mathrm{M}_2 = \mathrm{M}_2\mathrm{d}\mathrm{M}_1\mathrm{R} \tag{22}
$$

$$
\mathrm{R}\mathrm{d}\mathrm{M}_1 \wedge \mathrm{d}\mathrm{M}_2 = -\mathrm{d}\mathrm{M}_2 \wedge \mathrm{d}\mathrm{M}_1\mathrm{R}, \tag{23}
$$

where

$$
\mathrm{R} = \begin{pmatrix} 1 & -h & h & h^2 \\ 0 & 1 & 0 & -h \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad dM = \begin{pmatrix} \mathrm{d}a & \mathrm{d}b \\ \mathrm{d}c & \mathrm{d}d \end{pmatrix}. \tag{24}
$$

After applying the contraction, for the algebra of M_{ij} and $\mathrm{d}M_{ij}$ we obtain:

$$\begin{array}{ll}
\beta\mathrm{d}\gamma = \mathrm{d}\gamma\beta + \frac{1}{\mu}(\delta+\alpha)\mathrm{d}\gamma & \gamma\mathrm{d}\beta = \mathrm{d}\beta\gamma - \frac{1}{\mu}\gamma(\mathrm{d}\delta+\mathrm{d}\alpha) \\
\beta\mathrm{d}\alpha = \mathrm{d}\alpha\beta + \frac{1}{\mu}(\alpha-\delta)\mathrm{d}\alpha & \alpha\mathrm{d}\beta = \mathrm{d}\beta\alpha + \frac{1}{\mu}\alpha(-\mathrm{d}\alpha+\mathrm{d}\delta) \\
\beta\mathrm{d}\delta = \mathrm{d}\delta\beta + \frac{1}{\mu}(\delta-\alpha)\mathrm{d}\delta & \delta\mathrm{d}\beta = \mathrm{d}\beta\delta + \frac{1}{\mu}\delta(\mathrm{d}\alpha-\mathrm{d}\delta) \\
\multicolumn{2}{l}{\beta\mathrm{d}\beta = \mathrm{d}\beta\beta + (\frac{1}{\mu})^2(\delta\mathrm{d}\delta - \alpha\mathrm{d}\alpha) + \frac{1}{\mu}(\delta\mathrm{d}\beta - \beta\mathrm{d}\delta + \beta\mathrm{d}\alpha - \alpha\mathrm{d}\beta)}
\end{array} \tag{25}$$

All the remaining relations are commutative. After the contraction (23), results in;

$$\begin{array}{ll}
\mathrm{d}\alpha\wedge\mathrm{d}\alpha = \mathrm{d}\gamma\wedge\mathrm{d}\gamma = \mathrm{d}\delta\wedge\mathrm{d}\delta = 0 & \mathrm{d}\beta\wedge\mathrm{d}\beta = \frac{1}{2\mu}\ \{\mathrm{d}\beta\wedge(\mathrm{d}\delta+\mathrm{d}\alpha) - (\mathrm{d}\delta+\mathrm{d}\alpha)\wedge\mathrm{d}\beta\} \\
\mathrm{d}\alpha\wedge\mathrm{d}\beta + \mathrm{d}\beta\wedge\mathrm{d}\alpha = \frac{1}{\mu}\mathrm{d}\alpha\wedge\mathrm{d}\delta & \mathrm{d}\beta\wedge\mathrm{d}\gamma + \mathrm{d}\gamma\wedge\mathrm{d}\beta = \frac{1}{\mu}(\mathrm{d}\delta\wedge\mathrm{d}\gamma - \mathrm{d}\gamma\wedge\mathrm{d}\alpha) \\
\mathrm{d}\beta\wedge\mathrm{d}\delta + \mathrm{d}\delta\wedge\mathrm{d}\beta = \frac{1}{\mu}\mathrm{d}\delta\wedge\mathrm{d}\alpha & \mathrm{d}\alpha\wedge\mathrm{d}\gamma + \mathrm{d}\gamma\wedge\mathrm{d}\alpha = 0 \\
\mathrm{d}\delta\wedge\mathrm{d}\gamma + \mathrm{d}\gamma\wedge\mathrm{d}\delta = 0 & \mathrm{d}\alpha\wedge\mathrm{d}\delta + \mathrm{d}\delta\wedge\mathrm{d}\alpha = 0
\end{array} \tag{26}$$

3 Universal R matrix

A quasitriangular Hopf algebra is a Hopf algebra with a universal R matrix satisfying

$$\begin{array}{c}
(\Delta\otimes 1)R = R_{13}R_{23} \\
\\
(1\otimes\Delta)R = R_{13}R_{12}
\end{array} \tag{27}$$

$$R\Delta(\cdot)R^{-1} = \Delta'(\cdot) := \sigma\circ\Delta(\cdot) \tag{28}$$

where σ is the flip map: $\sigma(a\otimes b) = b\otimes a$. If in addition

$$\sigma(R^{-1}) = R \tag{29}$$

the Hopf algebra is called triangular [22].

Vladimirov [18] has considered the subalgebra of $\mathrm{U}_h\left(\mathsf{sl}(2)\right)$ generated by the two generators X and H and has found the following universal R matrix for this subalgebra.

$$\tilde{R} = \exp\left\{\frac{h\Delta X}{\sinh(h\Delta X)}\left[H\otimes\sinh(hX) - \sinh(hX)\otimes H\right]\right\} \tag{30}$$

Expressing this R matrix in terms of the new generators, and going to the $\epsilon\to 0$ limit, we get the following R matrix.

$$R = \exp(A) \quad A = 2\frac{\Delta\pi^+}{\sinh(\Delta\pi^+)}\left(K\otimes\sinh\pi^+ - \sinh\pi^+\otimes K\right). \tag{31}$$

We have used the following notation.

$$\pi^\pm := \mu^{-1}P^\pm \tag{32}$$

We will show that this is the universal R matrix for $\mathrm{U}_\mu\left(\mathsf{P}(2)\right)$. First, it is seen that this matrix is not singular. Second, as Vladimirov's R matrix, (30), satisfies the quantum Yang-Baxter equation, the relations (27), and also

$$\begin{aligned}\tilde{R}\Delta X\tilde{R}^{-1} &= \Delta' X\\ \tilde{R}\Delta H\tilde{R}^{-1} &= \Delta' H,\end{aligned}\tag{33}$$

it is obvious that this new R matrix also satisfies the quantum Yang-Baxter equation, the relations (27), and also

$$\begin{aligned}R\Delta\pi^+R^{-1} &= \Delta'\pi^+\\ R\Delta K R^{-1} &= \Delta' K.\end{aligned}\tag{34}$$

Now we want to prove that R is indeed the universal R matrix of the whole algebra $\mathrm{U}_\mu\left(\mathsf{P}(2)\right)$.

Define

$$E := R\Delta P^- R^{-1} - \Delta' P^-.\tag{35}$$

We are to prove that $E = 0$. To see this we use the commutation relation of P^- and K, and the fact that Δ and Δ' are homomorphisms of the algebra:

$$\begin{aligned}\left[R\Delta P^- R^{-1}, \Delta' K\right] &= R\Delta P^- R^{-1}\cosh(\Delta'\pi^+)\\ &= (E + \Delta' P^-)\cosh(\Delta'\pi^+)\end{aligned}\tag{36}$$

and,

$$\begin{aligned}\left[R\Delta P^- R^{-1}, \Delta' K\right] &= [E + \Delta' P^-, \Delta' K]\\ &= [E, \Delta' K] + \Delta' P^- \cosh(\Delta'\pi^+)\end{aligned}\tag{37}$$

which yields

$$[\Delta' K, E] = -E\cosh\Delta'\pi^+.\tag{38}$$

On the other hand, using Hausdorf's identity, we have

$$R\Delta P^- R^{-1} = \exp\left([A, \cdot\,]\right)\Delta P^-\tag{39}$$

It is easy to see that the commutator of A with a term like $P^- f(\pi^+)\otimes g(\pi^+)$ is a linear combination of other terms of this kind. Moreover, if f and g are analytic functions around $\pi^+ = 0$, the commutator is also analytic around $\pi^+ = 0$. Using these, one can write

$$E = C + D\tag{40}$$

where

$$\begin{aligned}C &:= \sum_{m,n\geq 0} C_{mn} P^-(\pi^+)^m \otimes (\pi^+)^n\\ D &:= \sum_{m,n\geq 0} D_{mn}(\pi^+)^m \otimes P^-(\pi^+)^n,\end{aligned}\tag{41}$$

and

$$\begin{aligned}
[\Delta' K, C] &= -C\cosh(\Delta'\pi^+) = -\left(C\cosh\pi^+\otimes\cosh\pi^+ + C\sinh\pi^+\otimes\sinh\pi^+\right) \\
[\Delta' K, D] &= -D\cosh(\Delta'\pi^+) = -\left(D\cosh\pi^+\otimes\cosh\pi^+ + D\sinh\pi^+\otimes\sinh\pi^+\right).
\end{aligned} \tag{42}$$

The first relation leads to

$$\sum_{m,n\geq 0} C_{mn}\left[\left(m\frac{\sinh\pi^+}{\pi^+} - \cosh\pi^+\right) P^-(\pi^+)^m \otimes (\pi^+)^n \exp(-\pi^+) + nP^-(\pi^+)^m \exp(\pi^+)\otimes(\pi^+)^n \frac{\sinh\pi^+}{\pi^+}\right]$$

$$= -\sum_{m,n\geq 0} C_{mn}\left[P^-(\pi^+)^m\cosh\pi^+\otimes(\pi^+)^n\cosh\pi^+ + P^-(\pi^+)^m\sinh\pi^+\otimes(\pi^+)^n\sinh\pi^+\right] \tag{43}$$

Now, suppose that at least one of the C_{mn}'s is nonzero. Then, it is easy to see that there exists a pair (m_0, n_0) such that

$$\begin{aligned}
&C_{m_0\, n_0} \neq 0 \\
&C_{mn} = 0 \qquad \text{if} \qquad \big(m \leq m_0, n \leq n_0, (m,n) \neq (m_0, n_0)\big)
\end{aligned} \tag{44}$$

Expanding the relation (43) in powers of π^+, and equating the coefficients of $P^-(\pi^+)^{m_0}\otimes(\pi^+)^{n_0}$ on both sides of the equation, one arrives at

$$(m_0 - 1 + n_0) C_{m_0\, n_0} = -C_{m_0\, n_0} \tag{45}$$

or,

$$(m_0 + n_0) C_{m_0\, n_0} = 0, \tag{46}$$

which yields

$$m_0 = n_0 = 0. \tag{47}$$

So, if one can show that $C_{00} = 0$, it implies that $C = 0$. The same argument is also true for D. this means that if E is zero up to zeroth order of π^+, it is zero to all orders of π^+. But we have

$$R|_{\pi^+=0} = 1 \qquad \Rightarrow \qquad R\Delta P^- R^{-1}|_{\pi^+=0} = 1\otimes P^- + P^-\otimes 1 = \Delta' P^-|_{\pi^+=0} \qquad \Rightarrow \tag{48}$$

$$E|_{\pi^+=0} = 0 \tag{49}$$

This completes the proof that this Hopf algebra is quasitriangular,

$$R\Delta P^- R^{-1} = \Delta' P^-. \tag{50}$$

Now it can be easily shown that (29) is indeed satisfied for the R matrix (31). This shows that $\mathrm{U}_\mu\,(\mathsf{P}(2))$ is triangular.

4 Deformation map

Following the idea of [23] one can find a deformation map for the $\mathrm{U}_\mu\left(\mathsf{p}(2)\right)$ which reads as follows:

$$\begin{aligned}
\pi^+ &:= \widehat{\pi}^+ \\
\pi^- &:= \frac{\sinh \widehat{\pi}^+}{\widehat{\pi}^+}\widehat{\pi}^- \\
K &:= \frac{\widehat{\pi}^+}{\sinh \widehat{\pi}^+}\widehat{K}
\end{aligned} \tag{51}$$

where $\widehat{\pi}^\pm$ and $\widehat{K}$ belong to the classical, i.e. undeformed, algebra. A simple calculation shows that the commutation relations (13) result from the commutation relations of the classical case. However, the co-product structure are not related in this way. This deformation map can be used to study the representation theory of $\mathrm{U}_\mu\left(\mathsf{p}(2)\right)$ as in the case of κ-Poincaré algebra [24]. A furthure remark is that, as in the case of κ-Poincaré algebra, the deformation map is singular for certain values of P^+ if μ is imaginary, because in this case the hyperbolic functions are replaced by trigonometric ones.

5 Mass-shells and finite boosts

The casimir of $\mathrm{U}_\mu\left(\mathsf{p}(2)\right)$ is

$$C_2 := \pi^- \sinh \pi^+ \tag{52}$$

To obtain finite boosts we postulate that K transforms any operator according to

$$\frac{\mathrm{d}\Omega}{\mathrm{d}\eta} = [K, \Omega]. \tag{53}$$

One can solve the system of differential equations

$$\begin{aligned}
\frac{\mathrm{d}\pi^+}{\mathrm{d}\eta} &= [K, \pi^+] = \sinh \pi^+ \\
\frac{\mathrm{d}\pi^-}{\mathrm{d}\eta} &= [K, \pi^-] = -\pi^- \cosh \pi^+
\end{aligned} \tag{54}$$

and get

$$\begin{aligned}
\tanh \pi^+ &= \exp(\eta) \tanh \pi_0^+ \\
(\pi^-)^2 + \frac{m^2}{\mu^2}\pi^- &= \exp(-2\eta)\left[(\pi_0^-)^2 + \frac{m^2}{\mu^2}\pi_0^-\right],
\end{aligned} \tag{55}$$

where m is the *mass*, i.e. $C_2 = m^2$. As one expects, the $\mu \to \infty$ limit of these are the familiar Lorentz transformation of energy and momentum in light cone coordinates. For imaginary μ, π^+ is an angle

variable. Figuers (1) and (2) show the mass-shells for real and imaginary μ. Note that, the topology of the light cone has changed in the case of imaginary μ. One can compare this with the κ-deformation of the Poincaré group [2], the Casimir of which reads

$$C_2 := 2\kappa \sinh^2(\frac{P_0}{2\kappa}) - P^2. \tag{56}$$

Finite boosts for the κ-Poincaré are worked out in [25]. The result is something in terms of Jacobi elliptic functions. A point is that, in four dimensional κ-Poincaré, one can set all the generators, except P_1, P_0, and L_1, equal to zero, and arrive to the two dimensional κ-Poincaré, which is the Lorentz counterpart of $E_l(2)$, introduced in [10]. It may be possible to find a deformation of the four dimensional Poincaré group containing, in the same fashion, $\mathrm{U}_\mu\left(\mathsf{p}(2)\right)$.

References

[1] O. Ogievetski, W. B. Schmidke, J. Wess, and B. Zumino; Lett. Mat. Phys. **23**, 233 (1991)

[2] J. Lukierski, A. Nowicki, H. Ruegg, and V. L. Tolstoy; Phys. Lett. **B 264**, 331 (1991)

[3] J. Lukierski, and A. Nowicki; Phys. Lett. **B 279**, 299 (1991)

[4] L. Castellani; Phys. Lett. **B279**, 291 (1992)

[5] A. Aghamohammadi, S. Rouhani, and A. Shariati; to appear in J. Phys. A

[6] M. Chaichian, and A. P. Demichev; Phys. Lett. **B304**, 220 (1993)

[7] L. L. Vaksman, and L. I. Korogodski; Sov. Math. Dokl. **39**, 173 (1989)

[8] S. L. Woronowicz; Lett. Math. Phys. **23**, 251 (1991)

[9] E. Celeghini, R. Giachetti, E. Sorace, and M. Tarlini; J. Math. Phys. **32**, 1159 (1991)

[10] A. Ballesteros, E. Celeghini, R. Giachetti, E. Sorace, and M. Tarlini; J. Phys. **A26**, 7495 (1993)

[11] E. E. Demidov, Yu. I. Manin, E. E. Mukhin, and D. V. Zhdanovich; Prog. Theor. Phys. Suppl. **102**, 203 (1990)

[12] M. Dubois-Violette, and G. Launer; Phys. Lett. **B245**, 175 (1990)

[13] S. Zakrzewski; Lett. Math. Phys. **22**, 287 (1991)

[14] Ch. Ohn; Lett. Math Phys. **25**, 85 (1992)

[15] B. A. Kupershmidt; J. Phys. **A25**, L1239 (1992)

[16] V. Karimipour; Lett. Math. Phys. **30**, 87 (1994)

[17] A. Aghamohammadi; Mod. Phys. Lett. **A8**, 2607 (1993)

[18] A. A. Vladimirov; Mod. Phys. Lett. **A8**, 2573 (1993)

[19] A. Aghamohammadi, M. Khorrami, and A. Shariati; IPM-94-61, hep-th/9410135

[20] A. Sudbery; Phys. Lett. **B 284**, 61 (1992)

[21] P. Schupp, P. Watts, and B. Zumino; Lett. Math. Phys. **25**, 139 (1992)

[22] S. Majid; Int. J. Mod. Phys. **A5**, 1 (1990)

[23] R. Curtright, C. Zachos; Phys. Lett. **B243**, 237 (1990)

[24] P. Maślanka; J. Math. Phys. **34**, 6025 (1993)

[25] J. Lukierski, H. Ruegg, and W. Rühl; Phys. Lett. **B313**, 357 (1993)